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Systematics of linearization of $N = 2$ nonlinear SUSY in superfield formulation

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Abstract

The relation between a $N = 2$ nonlinear supersymmetric (SUSY) model and a linear SUSY (free) theory for $N = 2$ vector supermultiplet accompanying the spontaneous SUSY breaking is systematically worked out in two-dimensional superfield formulation.

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Nonlinear (NL) realization of supersymmetry (SUSY) [1], which induces the spontaneous SUSY breaking, gives the way to construct NLSUSY general relativity (GR) [2, 3] as the fundamental theory of everything in the SGM scenario from a compositeness viewpoint [4, 5]. The low energy physics and cosmology in NLSUSY GR are discussed [5, 6, 7] based on the linearization of NLSUSY (*NL-linear(L) SUSY relation*) in (Riemann-)flat spacetime. The linearization problem in flat spacetime was addressed mainly so far for $N = 1$ and $N = 2$ SUSY by studying the relation between the NLSUSY model and various LSUSY *free* field theories (with the Fayet-Iliopoulos (FI) term), for $N = 1$ scalar supermultiplet [8]-[10], for $N = 1$ ($U(1)$) (axial) vector one [11] and for $N = 2$ ($SU(2) \times U(1)$) vector one [12]. Linearizing $N = 3$ NLSUSY was also discussed in two dimensional spacetime ($d = 2$) [13].

Recently, according to heuristic arguments, we have shown the explicit relation between the $N = 2$ NLSUSY model and $N = 2$ LSUSY *interacting* theories in $d = 2$, i.e. one with Yukawa interaction terms for the vector supermultiplet [14], and the other with $U(1)$ gauge interaction terms between the vector and the scalar supermultiplets ($N = 2$ SUSY QED) [15]. In order to further investigate the NL-L SUSY relation for $N \geq 2$ SUSY which is realistic in the SGM scenario, it is important to develop the systematic method of the linearization in superfield formulation [8, 10] into higher N SUSY theories. In this letter, as a preliminary to do this we discuss on the linearization of $N = 2$ NLSUSY ($N = 2$ NL-L SUSY relation) for the $N = 2$ vector supermultiplet in the $d = 2$ superfield formulation at the free-theory level.

In the linearization process, SUSY invariant relations connecting the NLSUSY model with a LSUSY theory are essential, where component fields in the LSUSY theory are expressed as composites in terms of Nambu-Goldstone (NG) fermion (*superon* in the SGM scenario). These relations are systematically obtained by defining a superfield on the following specific supertranslations [8, 10] of superspace coordinates (x^a, θ^i) depending on the (Majorana) NG fermions ψ^i, ‡

$$x'^a = x^a + i\kappa\bar{\theta}^i\gamma^a\psi^i,$$
$$\theta'^i = \theta^i - \kappa\psi^i, \tag{1}$$

where κ is a constant whose dimension is $(\text{mass})^{-1}$ in $d = 2$. Indeed, a superfield on (x'^a, θ'^i),

$$\tilde{\Phi}(x, \theta^i; \psi^i(x)) = \Phi(x', \theta'^i), \tag{2}$$

‡Minkowski spacetime indices are denoted by $a, b, \cdots = 0, 1$ in $d = 2$ and $SO(N)$ internal indices are $i, j, \cdots = 1, 2$ for $N = 2$. The Minkowski spacetime metric in $d = 2$ is $\frac{1}{2}\{\gamma^a, \gamma^b\} = \eta^{ab} = \text{diag}(+, -)$ and $\sigma^{ab} = \frac{i}{2}[\gamma^a, \gamma^b] = i\epsilon^{ab}\gamma_5$ ($\epsilon^{01} = 1 = -\epsilon_{01}$), where we use the γ matrices defined as $\gamma^0 = \sigma^2$, $\gamma^1 = i\sigma^1$, $\gamma_5 = \gamma^0\gamma^1 = \sigma^3$ with $\sigma^I (I = 1, 2, 3)$ being Pauli matrices.

transforms homogeneously as

$$\delta_\zeta \tilde{\Phi}(x, \theta^i) = \xi^a \partial_a \tilde{\Phi}(x, \theta^i) \tag{3}$$

with $\xi^a = i\kappa\bar{\psi}^i\gamma^a\zeta^i$, under superspace translations of (x^a, θ^i) accompanying NLSUSY transformations [1] of ψ^i,

$$\delta_\zeta \psi^i = \frac{1}{\kappa}\zeta^i - i\kappa\bar{\zeta}^j\gamma^a\psi^j\partial_a\psi^i, \tag{4}$$

parametrized by constant (Majorana) spinor parameters ζ^i. The supertransformation property (3) means that component fields $\tilde{\varphi}^I(x)$ in $\tilde{\Phi}(x, \theta^i)$ do not transform each other, and SUSY invariant constraints, $\tilde{\varphi}^I(x) = \text{constant}$, can be imposed, which leads to the SUSY invariant relations.

Let us introduce a $d = 2$, $N = 2$ (general) superfield [16, 17] for the $N = 2$ vector supermultiplet,

$$\begin{aligned}\mathcal{V}(x, \theta^i) = {} & C(x) + \bar{\theta}^i\Lambda^i(x) + \frac{1}{2}\bar{\theta}^i\theta^j M^{ij}(x) - \frac{1}{2}\bar{\theta}^i\theta^i M^{jj}(x) + \frac{1}{4}\epsilon^{ij}\bar{\theta}^i\gamma_5\theta^j\phi(x) \\ & -\frac{i}{4}\epsilon^{ij}\bar{\theta}^i\gamma_a\theta^j v^a(x) - \frac{1}{2}\bar{\theta}^i\theta^i\bar{\theta}^j\lambda^j(x) - \frac{1}{8}\bar{\theta}^i\theta^i\bar{\theta}^j\theta^j D(x),\end{aligned} \tag{5}$$

where the component fields are denoted by (C, D) for two scalar fields, (Λ^i, λ^i) for four spinor fields, ϕ for a pseudo scalar field, v^a for a vector field, and $M^{ij} = M^{(ij)}$ $\left(= \frac{1}{2}(M^{ij} + M^{ji})\right)$ for three scalar fields ($M^{ii} = \delta^{ij}M^{ij}$), respectively. The superfield (5) transforms under the superspace translations of (x^a, θ^i) as

$$\delta_\zeta \mathcal{V}(x, \theta^i) = \bar{\zeta}^i Q^i \mathcal{V}(x, \theta^i) \tag{6}$$

with supercharges

$$Q^i_\alpha = \frac{\partial}{\partial\bar{\theta}^{\alpha i}} + i\partial\!\!\!/\theta^i_\alpha, \tag{7}$$

satisfying $\{Q^i_\alpha, Q^j_\beta\} = -2\delta^{ij}(\gamma^a C)_{\alpha\beta}P_a$.

The $N = 2$ superfield (5) on the specific coordinates (1),

$$\tilde{\mathcal{V}}(x, \theta^i) = \mathcal{V}(x', \theta'^i), \tag{8}$$

may be expanded in component fields as

$$\tilde{\mathcal{V}}(x,\theta^i) = \tilde{C}(x) + \bar{\theta}^i\tilde{\Lambda}^i(x) + \frac{1}{2}\bar{\theta}^i\theta^j\tilde{M}^{ij}(x) - \frac{1}{2}\bar{\theta}^i\theta^i\tilde{M}^{jj}(x) + \frac{1}{4}\epsilon^{ij}\bar{\theta}^i\gamma_5\theta^j\tilde{\phi}(x)$$
$$-\frac{i}{4}\epsilon^{ij}\bar{\theta}^i\gamma_a\theta^j\tilde{v}^a(x) - \frac{1}{2}\bar{\theta}^i\theta^i\bar{\theta}^j\tilde{\lambda}^j(x) - \frac{1}{8}\bar{\theta}^i\theta^i\bar{\theta}^j\theta^j\tilde{D}(x), \tag{9}$$

where the component fields $\tilde{\varphi}^I(x) = (\tilde{C}(x), \tilde{\Lambda}^i(x), \tilde{M}^{ij}(x), \cdots)$ transform according to Eq.(3). The $\tilde{\varphi}^I(x)$ are evaluated in terms of $\varphi^I(x) = (C(x), \Lambda^i(x), M^{ij}(x)\cdots)$ in Eq.(5) as follows:

$$\begin{aligned}
\tilde{C} &= C' - \kappa\bar{\psi}^i\Lambda'^i + \frac{1}{2}\kappa^2(\bar{\psi}^i\psi^j M'^{ij} - \bar{\psi}^i\psi^i M'^{jj}) \\
&\quad + \frac{1}{4}\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j\phi' - \frac{i}{4}\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma_a\psi^j v'^a + \frac{1}{2}\kappa^3\bar{\psi}^i\psi^i\bar{\psi}^j\lambda'^j - \frac{1}{8}\kappa^4\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j D', \\
\tilde{\Lambda}^i &= \Lambda'^i - \kappa(\psi^j M'^{ij} - \psi^i M'^{jj}) - \frac{1}{2}\kappa\epsilon^{ij}\gamma_5\psi^j\phi' + \frac{i}{2}\kappa\epsilon^{ij}\gamma_a\psi^j v'^a \\
&\quad - \kappa^2\left(\psi^i\bar{\psi}^j\lambda'^j + \frac{1}{2}\bar{\psi}^j\psi^j\lambda'^i\right) + \frac{1}{2}\kappa^3\psi^i\bar{\psi}^j\psi^j D', \\
\tilde{M}^{ij} &= M'^{ij} - \kappa\bar{\psi}^{(i}\lambda'^{j)} + \frac{1}{2}\kappa^2\bar{\psi}^i\psi^j D', \\
\tilde{\phi} &= \phi' + \kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\lambda'^j - \frac{1}{2}\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j D', \\
\tilde{v}^a &= v'^a + i\kappa\epsilon^{ij}\bar{\psi}^i\gamma^a\lambda'^j - \frac{i}{2}\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j D', \\
\tilde{\lambda}^i &= \lambda'^i - \kappa\psi^i D', \\
\tilde{D} &= D',
\end{aligned} \tag{10}$$

where $\varphi'^I(x) = (C'(x), \Lambda'^i(x), M'^{ij}(x)\cdots)$ are the component fields in

$$\mathcal{V}(x',\theta'^i) = C'(x) + \bar{\theta}'^i\Lambda'^i(x) + \frac{1}{2}\bar{\theta}'^i\theta'^j M'^{ij}(x) - \frac{1}{2}\bar{\theta}'^i\theta'^i M'^{jj}(x) + \frac{1}{4}\epsilon^{ij}\bar{\theta}'^i\gamma_5\theta'^j\phi'(x)$$
$$-\frac{i}{4}\epsilon^{ij}\bar{\theta}'^i\gamma_a\theta'^j v^a(x) - \frac{1}{2}\bar{\theta}'^i\theta'^i\bar{\theta}'^j\lambda'^j(x) - \frac{1}{8}\bar{\theta}'^i\theta'^i\bar{\theta}'^j\theta'^j D(x), \tag{11}$$

and are expanded as

$$\begin{aligned}
C' &= C, \\
\Lambda'^i &= \Lambda^i + i\kappa\partial\!\!\!/ C\psi^i, \\
M'^{ij} &= M^{ij} - i\kappa\epsilon^{(i|k|}\epsilon^{j)l}\bar{\psi}^k\partial\!\!\!/\Lambda^l + \frac{1}{2}\kappa^2\epsilon^{ik}\epsilon^{jl}\bar{\psi}^k\psi^l\Box C, \\
\phi' &= \phi + i\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\partial\!\!\!/\Lambda^j - \frac{1}{2}\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j\Box C,
\end{aligned}$$

$$
\begin{aligned}
v'^a &= v^a + \kappa\epsilon^{ij}\bar{\psi}^i\partial\!\!\!/\gamma^a\Lambda^j - \frac{i}{2}\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j\Box C + i\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma^b\psi^j\partial^a\partial_b C, \\
\lambda'^i &= \lambda^i + i\kappa\partial\!\!\!/ M^{ij}\psi^j - \frac{i}{2}\kappa\epsilon^{ab}\epsilon^{ij}\gamma_a\psi^j\partial_b\phi + \frac{1}{2}\kappa\epsilon^{ij}\left(\psi^j\partial_a v^a - \frac{1}{2}\epsilon^{ab}\gamma_5\psi^j F_{ab}\right) \\
&\quad + \frac{1}{2}\kappa^2(\Box\Lambda^i\bar{\psi}^j\psi^j - \Box\Lambda^j\bar{\psi}^i\psi^j - \gamma_5\Box\Lambda^j\bar{\psi}^i\gamma_5\psi^j - \gamma_a\Box\Lambda^j\bar{\psi}^i\gamma^a\psi^j + 2\partial\!\!\!/\partial_a\Lambda^j\bar{\psi}^i\gamma^a\psi^j) \\
&\quad + \frac{i}{2}\kappa^3\partial\!\!\!/\Box C\psi^i\bar{\psi}^j\psi^j, \\
D' &= D + i\kappa\bar{\psi}^i\partial\!\!\!/\lambda^i \\
&\quad - \frac{1}{2}\kappa^2\left(\bar{\psi}^i\psi^j\Box M^{ij} - \frac{1}{2}\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j\Box\phi + \frac{i}{2}\epsilon^{ij}\bar{\psi}^i\gamma_a\psi^j\Box v^a - i\epsilon^{ij}\bar{\psi}^i\gamma_a\psi^j\partial_a\partial_b v^b\right) \\
&\quad + \frac{i}{2}\kappa^3\bar{\psi}^i\psi^i\bar{\psi}^j\partial\!\!\!/\Box\Lambda^j - \frac{1}{8}\kappa^4\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j\Box^2 C.
\end{aligned}
\tag{12}
$$

Solving Eq.(10) with respect to φ^I in terms of $(\tilde{\varphi}^I,\ \psi^i)$ and imposing SUSY (and gauge) invariant constraint on $\tilde{\lambda}^i$ can be considered as in refs.[8, 10], which leads to an action in terms of ψ^i interacting with other fields in $\tilde{\varphi}^I$, e.g. $\tilde{v}^a$.

However, focusing here on the sector which depends only on the NG fermions, we impose SUSY invariant constraints which eliminate the other degrees of freedom than ψ^i as, for example, the simplest ones,

$$
\tilde{C} = \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \quad \tilde{D} = \frac{\xi}{\kappa} \tag{13}
$$

with an arbitrary dimensionless parameter ξ. Then, from Eqs.(10) and (12) the relations between φ^I and ψ^i become

$$
\begin{aligned}
C &= -\frac{1}{8}\xi\kappa^3\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j, \\
\Lambda^i &= -\frac{1}{2}\xi\kappa^2\psi^i\bar{\psi}^j\psi^j - i\kappa\partial\!\!\!/ C\psi^i, \\
M^{ij} &= \frac{1}{2}\xi\kappa\bar{\psi}^i\psi^j + i\kappa\epsilon^{(i|k|}\epsilon^{j)l}\bar{\psi}^k\partial\!\!\!/\Lambda^l - \frac{1}{2}\kappa^2\epsilon^{ik}\epsilon^{jl}\bar{\psi}^k\psi^l\Box C, \\
\phi &= -\frac{1}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j - i\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\partial\!\!\!/\Lambda^j + \frac{1}{2}\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j\Box C, \\
v^a &= -\frac{i}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j - \kappa\epsilon^{ij}\bar{\psi}^i\partial\!\!\!/\gamma^a\Lambda^j + \frac{i}{2}\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j\Box C - i\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma^b\psi^j\partial^a\partial_b C, \\
\lambda^i &= \xi\psi^i - i\kappa\partial\!\!\!/ M^{ij}\psi^j + \frac{i}{2}\kappa\epsilon^{ab}\epsilon^{ij}\gamma_a\psi^j\partial_b\phi - \frac{1}{2}\kappa\epsilon^{ij}\left(\psi^j\partial_a v^a - \frac{1}{2}\epsilon^{ab}\gamma_5\psi^j F_{ab}\right) \\
&\quad - \frac{1}{2}\kappa^2(\Box\Lambda^i\bar{\psi}^j\psi^j - \Box\Lambda^j\bar{\psi}^i\psi^j - \gamma_5\Box\Lambda^j\bar{\psi}^i\gamma_5\psi^j - \gamma_a\Box\Lambda^j\bar{\psi}^i\gamma^a\psi^j + 2\partial\!\!\!/\partial_a\Lambda^j\bar{\psi}^i\gamma^a\psi^j)
\end{aligned}
$$

$$-\frac{i}{2}\kappa^3 \partial\!\!\!/ \Box C \psi^i \bar{\psi}^j \psi^j,$$

$$D = \frac{\xi}{\kappa} - i\kappa \bar{\psi}^i \partial\!\!\!/ \lambda^i$$
$$+\frac{1}{2}\kappa^2 \left(\bar{\psi}^i \psi^j \Box M^{ij} - \frac{1}{2}\epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j \Box \phi + \frac{i}{2}\epsilon^{ij} \bar{\psi}^i \gamma_a \psi^j \Box v^a - i\epsilon^{ij} \bar{\psi}^i \gamma_a \psi^j \partial_a \partial_b v^b \right)$$
$$-\frac{i}{2}\kappa^3 \bar{\psi}^i \psi^i \bar{\psi}^j \partial\!\!\!/ \Box \Lambda^j + \frac{1}{8}\kappa^4 \bar{\psi}^i \psi^i \bar{\psi}^j \psi^j \Box^2 C. \tag{14}$$

We solve Eq.(14) entirely with respect to the component fields φ^I as composites of the NG fermions ψ^i and we obtain SUSY invariant relations for the $d = 2$, $N = 2$ vector supermultiplet in all orders of ψ^i as follows:

$$\begin{aligned}
C &= -\frac{1}{8}\xi\kappa^3 \bar{\psi}^i \psi^i \bar{\psi}^j \psi^j, \\
\Lambda^i &= -\frac{1}{2}\xi\kappa^2 \psi^i \bar{\psi}^j \psi^j (1 - i\kappa^2 \bar{\psi}^k \partial\!\!\!/ \psi^k), \\
M^{ij} &= \frac{1}{2}\xi\kappa \bar{\psi}^i \psi^j \left(1 - i\kappa^2 \bar{\psi}^k \partial\!\!\!/ \psi^k - \frac{1}{2}\kappa^4 \epsilon^{ab} \bar{\psi}^k \psi^l \partial_a \bar{\psi}^k \gamma_5 \partial_b \psi^l \right), \\
\phi &= -\frac{1}{2}\xi\kappa \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j \left(1 - i\kappa^2 \bar{\psi}^k \partial\!\!\!/ \psi^k - \frac{1}{2}\kappa^4 \epsilon^{ab} \bar{\psi}^k \gamma_5 \psi^l \partial_a \bar{\psi}^k \partial_b \psi^l \right), \\
v^a &= -\frac{i}{2}\xi\kappa \epsilon^{ij} \bar{\psi}^i \gamma^a \psi^j (1 - i\kappa^2 \bar{\psi}^k \partial\!\!\!/ \psi^k), \\
\lambda^i &= \xi \psi^i |w|, \\
D &= \frac{\xi}{\kappa}|w|,
\end{aligned} \tag{15}$$

where $|w|$ is the determinant introduced in [1], which induces a spacetime-volume differential form in the NLSUSY model, i.e. for the $d = 2$, $N = 2$ ($N > 2$, as well) SUSY case,

$$|w| = \det(w^a{}_b) = \det(\delta^a_b + t^a{}_b), \quad t^a{}_b = -i\kappa^2 \bar{\psi}^i \gamma^a \partial_b \psi^i, \tag{16}$$

expanded in terms of $t^a{}_b$ or ψ^i as

$$\begin{aligned}
|w| &= 1 + t^a{}_a + \frac{1}{2!}(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a) \\
&= 1 - i\kappa^2 \bar{\psi}^i \partial\!\!\!/ \psi^i - \frac{1}{2}\kappa^4 (\bar{\psi}^i \partial\!\!\!/ \psi^i \bar{\psi}^j \partial\!\!\!/ \psi^j - \bar{\psi}^i \gamma^a \partial_b \psi^i \bar{\psi}^j \gamma^b \partial_a \psi^j) \\
&= 1 - i\kappa^2 \bar{\psi}^i \partial\!\!\!/ \psi^i - \frac{1}{2}\kappa^4 \epsilon^{ab} (\bar{\psi}^i \psi^j \partial_a \bar{\psi}^i \gamma_5 \partial_b \psi^j + \bar{\psi}^i \gamma_5 \psi^j \partial_a \bar{\psi}^i \partial_b \psi^j).
\end{aligned} \tag{17}$$

Note that all SUSY invariant relations for φ^I in Eq.(15) are expressed as the form,

$$\varphi^I \sim \xi \kappa^{n-1} (\psi^i)^n |w| \ (n = 0, 1, \cdots, 4), \tag{18}$$

where $(\psi^i)^2$ means $\bar{\psi}^i \psi^j$, $\epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j$ or $\epsilon^{ij} \bar{\psi}^i \gamma^a \psi^j$, $(\psi^i)^3 = \psi^i \bar{\psi}^j \psi^j$ and $(\psi^i)^4 = \bar{\psi}^i \psi^i \bar{\psi}^j \psi^j$.

Let us now discuss on the relation between NLSUSY and LSUSY actions for the $N = 2$ vector supermultiplet in the free theory. The NLSUSY action [1] for $d = 2$, $N = 2$ SUSY is written in terms of ψ^i as

$$S_{N=2\mathrm{NLSUSY}} = -\frac{1}{2\kappa^2} \int d^2x \ |w|, \tag{19}$$

which is invariant (becomes a surface term) under the NLSUSY transformations (4) due to $\delta_\zeta |w| = \partial_a (\xi^a |w|)$. On the other hand, the (free) action for the $N = 2$ vector supermultiplet with the FI D term is given by using the superfield (5) as follows:

$$S_{\mathcal{V}0} = \int d^2x \left[\int d^2\theta^i \mathcal{L}_0(x, \theta^i) + \int d^4\theta^i \mathcal{L}_{\mathrm{FI}}(x, \theta^i) \right]_{\theta^i = 0}, \tag{20}$$

where

$$\mathcal{L}_0(x, \theta^i) = \frac{1}{32} (\overline{D^j \mathcal{W}^{kl}} D^j \mathcal{W}^{kl} + \overline{D^j \mathcal{W}_5^{kl}} D^j \mathcal{W}_5^{kl}), \tag{21}$$

$$\mathcal{L}_{\mathrm{FI}}(x, \theta^i) = \frac{\xi}{2\kappa} \mathcal{V} \tag{22}$$

with

$$D_\alpha^i = \frac{\partial}{\partial \bar{\theta}^{\alpha i}} - i \partial\!\!\!/ \theta_\alpha^i, \tag{23}$$

$$\mathcal{W}^{ij} = \bar{D}^i D^j \mathcal{V}, \quad \mathcal{W}_5^{ij} = \bar{D}^i \gamma_5 D^j \mathcal{V}. \tag{24}$$

The action (20) in the WZ gauge gives the $N = 2$ LSUSY (free) action for the minimal off-shell component fields $(A, \phi, v^a, \lambda^i, D)$ with $A = M^{ii} (= M^{11} + M^{22})$ [17],

$$\begin{aligned} S_{V0} &= S_{\mathcal{V}0} \ |_{\mathrm{WZ\ gauge}} \\ &= \int d^2x \left\{ -\frac{1}{4}(F_{ab})^2 + \frac{i}{2} \bar{\lambda}^i \partial\!\!\!/ \lambda^i + \frac{1}{2} (\partial_a A)^2 + \frac{1}{2} (\partial_a \phi)^2 + \frac{1}{2} D^2 - \frac{\xi}{\kappa} D \right\}, \end{aligned} \tag{25}$$

where the field equation for the auxiliary field, $D = \frac{\xi}{\kappa}$, indicates the spontaneous SUSY breaking.

The relation between the actions (19) and (25) with $\xi^2 = 1$, i.e.

$$S_{N=2\mathrm{NLSUSY}} = S_{V0} + [\text{suface terms}], \tag{26}$$

can be shown by substituting SUSY invariant relations for the minimal off-shell vector supermultiplet, $(A, \phi, v^a, \lambda^i, D)(\psi^i)$, into the action (25) directly [14]. Here let us show that the LSUSY action (20) exactly reduces to the NLSUSY action (19) when $\xi^2 = 1$ by using the superfield (9) in the SUSY invariant constraints (13),

$$\tilde{\mathcal{V}}(x, \theta^i) = -\frac{\xi}{8\kappa}\bar{\theta}^i\theta^i\bar{\theta}^j\theta^j, \tag{27}$$

which lead to the SUSY invariant relations (15): Indeed, by changing the integration variables in Eq.(20) from (x, θ^i) to (x', θ'^i), we obtain

$$\begin{aligned} S_{\mathcal{V}0} &= \int d^2x' \left[\int d^2\theta'^i \mathcal{L}_0(x', \theta'^i) + \int d^4\theta'^i \mathcal{L}_{\mathrm{FI}}(x', \theta'^i)\right]_{\theta'^i=0} \\ &= \int d^2x \left[\int d^2\theta^i J(x, \theta^i)\tilde{\mathcal{L}}_0(x, \theta^i) + \int d^4\theta^i J(x, \theta^i)\tilde{\mathcal{L}}_{\mathrm{FI}}(x, \theta^i)\right]_{\theta^i=0}, \end{aligned} \tag{28}$$

where

$$\tilde{\mathcal{L}}_0(x, \theta^i) = \frac{1}{32}(\overline{D'^j\tilde{\mathcal{W}}^{kl}}D'^j\tilde{\mathcal{W}}^{kl} + \overline{D'^j\tilde{\mathcal{W}}_5^{kl}}D'^j\tilde{\mathcal{W}}_5^{kl}), \tag{29}$$

$$\tilde{\mathcal{L}}_{\mathrm{FI}}(x, \theta^i) = \frac{\xi}{2\kappa}\tilde{\mathcal{V}} \tag{30}$$

with

$$D'^i_\alpha = \frac{\partial}{\partial\bar{\theta}'^{\alpha i}} - i\partial\!\!\!/' \theta'^i_\alpha, \tag{31}$$

$$\tilde{\mathcal{W}}^{ij} = \bar{D}'^i D'^j \tilde{\mathcal{V}}, \quad \tilde{\mathcal{W}}_5^{ij} = \bar{D}'^i \gamma_5 D'^j \tilde{\mathcal{V}}. \tag{32}$$

In Eq.(28) the $J(x, \theta^i)$ means the Jacobian given by

$$J(x, \theta^i) = \mathrm{sdet}M = |w| \det(\delta^a_b - i\kappa\nabla_b\bar{\psi}^i\gamma^a\theta^i), \tag{33}$$

where sdet is the superdeterminant, the supermatix M and the "covariant" derivative ∇_a [10] are defined by

$$\begin{aligned} M &= \frac{\partial(x', \theta'^i)}{\partial(x, \theta^j)} = \begin{pmatrix} \delta^a_b - i\kappa\partial_b\bar{\psi}^i\gamma^a\theta^i & -\kappa\partial_b\bar{\psi}^i \\ i\kappa\gamma^a\psi^j & \delta^{ij} \end{pmatrix}, \\ \nabla_a &= (w^{-1})_a{}^b\partial_b. \end{aligned} \tag{34}$$

Also, the transformation of derivatives is

$$\begin{pmatrix} \partial'_a \\ \frac{\partial}{\partial \theta'^i} \end{pmatrix} = M^{-1} \begin{pmatrix} \partial_a \\ \frac{\partial}{\partial \theta^i} \end{pmatrix}, \quad M^{-1} = \begin{pmatrix} {v_a}^b & \kappa {v_a}^b \partial_b \bar{\psi}^j \\ -i\kappa \gamma^a \psi^i {v_a}^b & \delta^{ij} - i\kappa^2 \gamma^a \psi^i \partial_b \bar{\psi}^j {v_a}^b \end{pmatrix}, \tag{35}$$

where ${v_a}^b$ is determined from

$$ {w_a}^c (\delta^d_c - i\kappa \nabla_c \bar{\psi}^i \gamma^d \theta^i) \ {v_d}^b = \delta^b_a, \tag{36}$$

and is solved as

$$ {v_a}^b = (w^{-1})_a{}^b + i\kappa \nabla_a \bar{\psi}^i \gamma^c \theta^i (w^{-1})_c{}^b + \mathcal{O}((\theta^i)^2). \tag{37}$$

Then the differential operators (31) are expressed by means of $\left(\partial_a, \frac{\partial}{\partial \theta^i}\right)$ as

$$ D'^i_\alpha = \frac{\partial}{\partial \bar{\theta}^{\alpha i}} - i\gamma^a \theta^i_\alpha {v_a}^b \left(\partial_b + \kappa \partial_b \bar{\psi}^j \frac{\partial}{\partial \bar{\theta}^{\alpha j}} \right), \tag{38}$$

By substituting Eqs.(27), (33) and (38) into Eq.(28), the relation between the actions (19) and (20),

$$ S_{N=2\mathrm{NLSUSY}} = S_{\mathcal{V}0}, \tag{39}$$

is shown when $\xi^2 = 1$.

We summarize our results as follows. In this letter we have systematically linearized $N = 2$ NLSUSY in the $d = 2$ superfield formulation for the $N = 2$ vector supermultiplet. Based on the $d = 2$, $N = 2$ superfield (5) the relation between the component fields $\tilde{\varphi}^I(x)$ in Eq.(9) and $\varphi^I(x)$ in Eq.(5) are given as in Eqs.(10) and (12). By imposing the (simplest) SUSY invariant constraints (13), we have obtained the SUSY invariant relations (15) uniquely, which coincide with those for the minimal off-shell vector supermultiplet obtained heuristicly in Ref.[14]. The $N = 2$ NLSUSY action (19) is just reproduced when $\xi^2 = 1$ by substituting the SUSY invariant relations into the $N = 2$ LSUSY free action with the FI D term (20), i.e. we have shown the relation (39) in the free theory from the superfield formulation. The extensions of the superfield method for the linearization to higher N NLSUSY and to $d = 4$ are important. The Yukawa interaction terms [14] and the coupling of matter supermultiplets (SUSY QED) [15] in the linearization framework of this letter are interesting problems under the investigation.

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